Drop Call Probability in Established Cellular Networks: from data Analysis to Modelling

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Abstract—The ever increase of advanced services offered by modern cellular networks require stringent Quality of Service (QoS) guarantee, obtained as the typical result of many optimization procedures. In this paper the phenomenon of dropped calls, one of the most important indices of QoS in a large scale well-established cellular network, has been analyzed. We verified from measured data traffic that, in a well-established cellular network, models available in literature are useless to pursue the objective of service optimization: many phenomena, neglected till now, heavily influence the call termination. To relate the drop call probability to these phenomena, an original analytical model has been developed. The obtained results, validated by experimental measures taken from a real network, can allow the network operator to optimize system performance improving the offered Quality of Service and their revenue.

I. INTRODUCTION

The drop call probability is one of the most important Quality of Service (QoS) index used to monitor the performance of cellular networks. Its minimization is the typical result of many optimization procedures applied to several service aspects as: the maximization of service coverage area and of network usage, the minimization of interference and congestion, the optimum traffic balancing among the different frequency layers (e.g., 900 and 1800 MHz in the European GSM standard).

For these reasons, the drop call probability has been the subject of several studies [1] - [8]. Generally, in the models developed in these papers, appropriate cell and system planning, taking into account propagation conditions in the service area, are considered. Moreover, the fundamental hypothesis that calls are terminated only due to handover failure is assumed (see for example, [2], [7]); that is, an active user can change cell several times and, during one of these transitions between cells, his connection can be dropped. Call termination caused by handover failure depends, apart from system characteristics, on user behaviour; therefore, it is an event non directly predictable by system planners.

The evolution of these models concerns the release of some restrictive hypotheses, such as a more general modelling of Call Holding Time or Cell Residence Time, initially assumed exponentially distributed [4], [6], [9], or the classification of users in different classes to take into account different mobility and services [10], or a different modelling of the cells assumed as service centers with a finite user population [11].

Although there is a vast literature on this subject, the analysis of measured data traffic, supplied by Vodafone Italy

and covering a part of the southern Italian territory, allowed us to verify that, in a well-established cellular network, previous models are useless to pursue the objective of service optimization. In fact, observing the behavior of actual wireless cellular networks, the handover failure is usually a negligible event. So that, many other phenomena become much more relevant in influencing the call dropping; in particular, call termination is mainly due to propagation conditions, irregular user behavior, and so on.

Modelling these phenomena is quite interesting both from theoretic and practical point of views. In fact, the knowledge of call dropping behaviour as a function of some network parameters (e.g., traffic load, call duration, and so on) can help operators to optimize performance and then to improve QoS and their revenue.

Therefore, in this paper a simple new drop call probability model, which is integrated in an experimental context, has been developed, with specific attention to the model validation on a real GSM network. The data traffic have been exploited in two ways. First we studied the call duration statistic distributions, the phenomena that lead to call termination, the characterization of both call arrival process and call drop process. As a second aspect, these data drive us in formulating the analytical model and were useful for a comparison with the results obtained from the proposed model. Even if we validated the model with GSM data, the approach is more general. Following a similar procedure, model parameters can be easily derived from measured data in other cellular systems (e.g., UMTS cellular networks) and, then, applied to performance prediction.

The rest of the paper is organized as follows. In section II the supplied measured data have been statistically analyzed. In section III the original analytical model has been developed evaluating the drop call probability, as a function of some network parameters. Numerical results are reported in section IV. Finally, some conclusions are drawn in section V.

II. DATA COLLECTION AND ANALYSIS

In this paper, several data have been collected for ten months from the Vodafone Italy GSM network: from February 2003 to December 2003. These data sets were related to ten GSM traffic cells, for a total of about 200,000 monitored calls, chosen as a representative sample to obtain numerically significant data. The sample significance is defined in terms of cell extension, number of served subscribers in the area,

TABLE I.
ESTIMATED STATISTICAL PARAMETERS

		$\hat{\mu}$ [s]	$\hat{\sigma}$ [s]	C
Cell 1	Normal Call Duration	121.74	205.65	1.69
	Dropped Call Duration	96.01	172.09	1.79
	Inter-departure time	92.44	87.67	0.95
	Interarrival time	6.14	6.14	1.00
Cell 2	Normal Call Duration	93.20	152.18	1.63
	Dropped Call Duration	130.20	339.70	2.61
	Inter-departure time	67.72	78.23	1.16
	Interarrival time	6.60	6.54	0.99
Cell 3	Normal Call Duration	100.97	134.89	1.34
	Dropped Call Duration	92.86	159.35	1.72
	Inter-departure time	101.08	103.33	1.02
	Interarrival time	3.18	3.53	1.11
Cell 4	Normal Call Duration	111.15	187.50	1.69
	Dropped Call Duration	95.64	213.47	2.23
	Inter-departure time	85.01	94.28	1.11
	Interarrival time	6.54	7.01	1.07
Cell 5	Normal Call Duration	108.35	198.13	1.83
	Dropped Call Duration	97.27	174.25	1.79
	Inter-departure time	99.65	101.27	1.01
	Interarrival time	4.00	5.00	1.25

and traffic load; for this reasons, macro GSM cells in a urban metropolitan environment where chosen.

By using the *Clear Codes* reported in the databases of the network operator, calls are classified in *Dropped* and *Not Dropped*, distinguishing for the former the causes of dropping. By observing the *Dropped* and *Not Dropped Calls*, and the difference between their starting instants, it is possible to evaluate the distribution of the arrival process, which we refer to as *interarrival process*. Likewise, from the analysis of the difference between the ending times, it is possible to evaluate the distribution of *inter-departure* times for *Dropped Calls*. Moreover, we have analyzed the durations of normally terminated calls and dropped calls calculated as difference between the charging ending time.

First of all, a statistical analysis of the measured data was carried out, allowing us the estimation of mean μ and variance σ^2 of each process, by using the well known convergent and not polarized estimators for these parameters [12]. Moreover, we evaluate the coefficient of variation, C , defined as the ratio between standard deviation and mean; this parameter is an index of data dispersion around the mean value. In Table I estimated statistical parameters referred to 4 hours around the Busy Hour (i.e., the hour in which there is the maximum traffic load in the network) are reported for some cells. Given that the coefficient of variation takes into account the first and second order sample statistic, it allows us to make a first choice between the candidate statistical distribution to fit measured data. In particular, we observed that the durations of normally terminated calls and dropped calls show a value of C greater than 1, whereas the interarrival and the inter-departure times have a coefficient of variation $C \simeq 1$.

From this and, by taking also into account other known results about the lognormal distribution of the channel holding time (i.e., the time during which a channel in a cell is occupied

by a call) [13], we made the hypothesis that call durations have a lognormal distribution whereas interarrival and inter-departure times have an exponential distribution. The analytical expressions of exponential and lognormal distribution are reported below [12]:

$$f_X(x) = \lambda e^{-\lambda x}, \quad \lambda > 0, x \geq 0, \quad (1)$$

$$g_T(t) = \frac{1}{t\varphi\sqrt{2\pi}} e^{-\frac{\ln t - \vartheta}{\varphi^2}}, \quad \varphi, \vartheta > 0, t \geq 0. \quad (2)$$

It is worthwhile to note that this result is different from the one reported in [13], where a lognormal is used to fit the channel holding time for a single cell, while the call duration, considered in this paper, is the sum of all the channel holding times in the cells visited by the user during the same call.

To check these hypotheses, the chi-square goodness-of-fit test (or χ^2 -test) was used [12], [13], following the algorithm introduced by Fisher [14]. For the estimation of distribution parameters, the corresponding maximum likelihood estimators were used for both the exponential and lognormal distribution, which are respectively (see [12], [13]):

$$\hat{\lambda} = n / \sum_{i=1}^n T_i, \quad (3)$$

$$\hat{\vartheta} = \frac{1}{n} \sum_{i=1}^n \ln(t_i), \quad \hat{\varphi} = \frac{1}{n} \sum_{i=1}^n \ln(t_i)^2 - \hat{\vartheta}^2, \quad (4)$$

where T_i and t_i are the time samples.

The tests were done using a significance level $\alpha = 0.01$. They gave positive results in all the trials, both with cell data referred to 24 hours and with data for the same cell referred to 4 hours around the Busy Hour. As in [13], also in this work it was necessary to filter data samples which showed an anomalous relative frequency, but, whereas in [13] the 26% of the sample data were reject, in our analysis we never discard more than 5% of data.

The obtained results show that both durations of normally terminated calls and dropped calls are lognormal distributed. Moreover, our statistical analysis confirms the exponential hypothesis both for call arrival process and for the inter-departure process between consecutive dropped calls. Even if these results confirms some previous similar results [13], we highlight, however, that the analysis of dropped calls for both interdeparture time and call duration time does not appear in previous scientific papers.

As an example, in Fig. 1 the measured samples and the fitted lognormal distribution for the duration of normal terminated calls are reported, while in Fig. 2 the interarrival times between calls, are fitted by an exponential distribution.

In the analysis of experimental measures, it was studied the aforesaid dropped calls causes by means of the *Clear Codes* and by the correspondence tables between these codes and the related physical phenomena. In Table II, results for one cell are illustrated. The analysis was repeated referring to different periods of time and to different cells to exclude

Fig. 1. Fitting of call duration with lognormal distribution (cell 1 observed for 4 hours).

Fig. 2. Fitting of interarrival time between calls, with exponential distribution (cell 1 observed for 4 hours).

possible influence of local phenomena. The prevailing causes are the electromagnetic ones (e.g., power attenuation, deep fading). A lot of calls are dropped due to irregular user behaviour (e.g., mobile equipment failure, phone switched off after ringing, subscriber charging capacity exceeded during call). Other causes are due to abnormal network response (e.g., radio and signalling protocols error).

The main conclusion of drop cause analysis was that, in a well-established cellular network, was not possible to find a prevailing cause of call forced termination, rather a heterogeneous independent mix of causes. In particular, the handover failure is almost an unknown event in such environment thanks to the reliability and the effectiveness of the deployed handover control procedure. Our opinion is that the obtained results are not limited to the analyzed cells, but generally valid on the whole network.

The effort made in this paper was to model all the causes

TABLE II.
OCCURRENCE OF CALL DROPPING CAUSES

DROP CAUSES	OCCURRENCE [%]
Electromagnetic causes	51.4
Irregular user behaviour	36.9
Abnormal network response	7.6
Others	4.1

of forced termination as a single phenomenon following the statistic of Poisson. In fact, thanks to the aforesaid exponential distribution of both the interarrival and departure times, and with the additional hypothesis of their independence, the Poissonian hypothesis for either the call arrival and call dropping process can be accepted.

III. ANALYTICAL MODEL

Starting from data analysis, we have developed an analytical model to predict the drop call probability as a function of network parameters.

Let λ_t be the total traffic entering in the generic cell. If the call blocking probability due to absence of available traffic channels, P_b , is almost negligible (that is, the system can be considered as non blocking), then λ_t is also the total traffic accepted in the cell. A fraction of this traffic will be dropped with probability P_d , that is the drop call probability, by phenomena described in the previous section; therefore the intensity ν_d of the Poisson process which models the drop call behaviour is given by:

$$\nu_d = \lambda_t \cdot P_d. \quad (5)$$

Now, we can consider the generic call behaviour (see Fig. 3): a call request is served by a generic channel, randomly selected, and the call will finish, if correctly terminated, after a duration time, T , which is a random variable with a lognormal p.d.f, $g_c(t)$, as we shown in the previous section. Otherwise, if one of the phenomena, which cause the call dropping, happens, the call will be dropped after a duration time, T_d , which is also a random variable, with a lognormal p.d.f., $g_d(t)$. Given that a Poisson process models the calls arrival, the cell can be modelled as a M/G/ ∞ queue.

T : lognormal distributed random variable

Fig. 3. Time diagram to describe cell behaviour.

The probability that a call, among k active ones, is not involved by one single drop event, during the duration time $T = t$, is $(k - 1) / k$; obviously, given that drop events are independent, this probability is $[(k - 1) / k]^n$ if there are n drop events. On the other hand, the inter-departure times,

which are the drop call instants, constitute a Poisson process with parameter ν_d . Let Y be the random variable that counts the number of drops; thus, the probability that there are n drops in $T = t$ is:

$$P(Y = n) = \frac{(\nu_d t)^n}{n!} e^{-\nu_d t}, \quad n \geq 0. \quad (6)$$

Hence, by the total probability theorem, the probability $P(C|T = t, N = k)$ that a call is completed, with duration $T = t$ and when in the presence of k contemporary calls, is:

$$\begin{aligned} P(C|T = t, N = k) &= \sum_{n=0}^{\infty} \left(\frac{k-1}{k} \right)^n \frac{(\nu_d t)^n}{n!} e^{-\nu_d t} dt \\ &= e^{-\nu_d t} \sum_{n=0}^{\infty} \frac{\left(\frac{k-1}{k} \nu_d t \right)^n}{n!} \\ &= e^{-\nu_d t} \cdot e^{\frac{k-1}{k} \nu_d t} = e^{-\frac{\nu_d t}{k}}. \end{aligned} \quad (7)$$

Using again the total probability theorem, summing over all the possible numbers of contemporary active calls, the probability that a call is normally terminated with duration t is:

$$P(C|T = t) = \sum_{k=1}^{\infty} P(C|T = t, N = k) \cdot P(N = k) \quad (8)$$

where $P(N = k)$ is the probability that in the cell there are k active users.

As experimentally verified, in well-established cellular networks, operating in normal conditions, the dropping causes are not due to unavailability of communication channels. As a consequence, a generic call can be modelled as belonging to a M/G/ ∞ queue. From queueing theory we have [15]:

$$P(N = k) = c_N \cdot \frac{\rho^k}{k!}, \quad k \geq 1, \quad (9)$$

where ρ is the utilization factor, given by the product between the total traffic λ_t and the mean service time T_c , and c_N is a normalization coefficient which considers that there is at least one ongoing call during the time $T = t$.

Applying the normalization condition it follows that the coefficient c is given by:

$$c_N = \frac{1}{e^{\rho} - 1}, \quad (10)$$

and substituting (10) in (9), we obtain:

$$P(N = k) = \frac{1}{e^{\rho} - 1} \cdot \frac{\rho^k}{k!}, \quad k \geq 1. \quad (11)$$

Now, substituting (11) and (7) in (8), we have:

$$P(C|T = t) = \sum_{k=1}^{\infty} e^{-\frac{\nu_d t}{k}} \cdot \frac{1}{e^{\rho} - 1} \cdot \frac{\rho^k}{k!}. \quad (12)$$

Thus, it is straightforward to evaluate the probability of a normally terminated call, P_c , simply considering every possible call duration:

$$P_c = \int_0^{\infty} P(C|T = t) g_c(t) dt = c_N \sum_{k=1}^{\infty} \frac{\rho^k}{k!} \int_0^{\infty} g_c(t) e^{-\frac{\nu_d t}{k}} dt. \quad (13)$$

TABLE III.
DROP CALL PROBABILITY RESULTS

	(by measures) P_d [%]	(by model) P_d [%]	Confidence Interval [%]
Cell 1	6.79	6.52	[5.84; 7.88]
Cell 2	7.29	7.47	[6.27; 8.46]
Cell 3	3.07	3.12	[2.61; 3.61]
Cell 4	6.72	6.74	[5.75; 7.84]
Cell 5	4.04	4.00	[3.44; 4.74]

Finally, since the drop call probability is $P_d = (1 - P_c)$, it results that:

$$P_d = 1 - \frac{1}{e^{\rho} - 1} \sum_{k=1}^{\infty} \frac{\rho^k}{k!} \int_0^{\infty} g_c(t) e^{-\frac{\nu_d t}{k}} dt. \quad (14)$$

It is worth noticing that eq. (14) depends on the drop call rate ν_d , the p.d.f. $g_c(t)$ of the duration of normally terminated calls, the average number ρ of active users in the cell, and the total call arrival rate λ_t .

Finally, given that in developing our model there are no assumption on a particular technology, it is worthwhile to note that the model can be exploited to predict the drop call probability in different cellular networks (e.g., GSM, PCS, UMTS).

IV. RESULTS

In this section numerical results obtained with the developed model are reported. First, for each examined cell, we evaluated, from the measured data, an estimation of the call drop probability P_d , and its confidence interval with an usual confidence level $1 - \alpha = 0.95$ [12]. This is to establish the acceptance region for the analytical results, so that values coming from the analytical model are acceptable provided that they fall in that interval.

Then, the model has been validated by comparing drop call probability estimated by measured data with the ones evaluated with the developed analytical model. In Table III such a comparison is reported for some cells and it shows that in every case the analytical results fall in the call drop probability confidence interval. We found that this behaviour is verified for all sets of measured data. The results are more accurate for data sets referred to 4 hours around the Busy Hour, which nevertheless are the most interesting for network operator purpose.

A better agreement comparing drop call probabilities evaluated by model and by measured data could be obtained by using larger data sample [12]. In fact, as the data set get larger (for heavy traffic cells) both the confidence interval get smaller and the estimation of the analytical model input parameter (ν_d , λ_t , etc.) get more accurate, reducing errors coming out from the data statistical fluctuation.

The model can be exploited to evaluate network performance as a function of network parameters. We verified the correlation between ν_d and λ_t , i.e., if variation of λ_t results in

ν_d variation. From data we found out that a linear dependence law between these two parameters exists:

$$\nu_d = m\lambda_t + b, \quad (15)$$

where m and b could be obtained with a least square regression technique [12]. Fig.4 shows that relatively great λ_t variations produce only little ν_d changes.

Fig. 4. Total traffic entering a cell, λ_t , vs. the drop call rate, ν_d .

In Fig. 5 the drop call probability is reported as a function of the total traffic entering in the cell, λ_t , with several values of coefficient of variation. The mean call duration is assumed equal to 100 s, near to the typical value observed in measured data (see Table I).

As shown in Fig.5, system performance improves as the traffic entering in the cell increases. Since ν_d varies according to eq. (15), increasing the traffic load, the number of dropped calls remains quite constant and, then, the drop call rate decreases.

The more interesting result coming out from Fig. 5 is the effect of coefficient of variation on drop call probability. This probability decreases as coefficient of variation increases; that is, fixing mean call duration, values more dispersed around this mean reduce call dropping probability. Similar results on other system performance are reported in literature [6] [16]. This behaviour can partially explain the performance improvement of some well-established mobile networks in which the presence at the same time of many services leads to a greater differentiation of call durations.

Fig. 5. Drop call probability vs traffic λ_t , with several coefficients of variation C .

V. CONCLUSIONS

In this paper, starting from the statistical analysis of data in a real cellular network, we have verified the lognormal hypothesis for distribution of the call holding time of both the normally terminated and the dropped calls. Moreover, the phenomena which cause the conversation interruption have been classified, verifying that handover failure become negligible in a well-established mobile telecommunication network. With both planning optimization and fine tuning of network parameters, several secondary phenomena (irregular users behaviours, abnormal network response, power attenuation, and so on) become non negligible and have been analytically modelled. The proposed new model has been validated comparing its results with data collected from a real cellular network, in a wide range of traffic load conditions.

The approach adopted in this paper can be fruitfully exploited to study different cellular networks with different distributions for the call duration, provided that eq. (14) can be numerically solved.

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